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Ethnomathematics and the responsible subversion of its pedagogical action: an investigation based on three anthropological approaches

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Abstract

This theoretical article primarily reflects upon the many points of view from an ongoing research in ethnomathematics and its connections to the responsible subversion of its pedagogical action. Furthermore, there is a discussion on mathematical ideas, procedures, and practices related to the anthropological approaches — emics (local/insiders) and etics (global/outsiders) — in order to understand the dialogic approach (emic-etic/glocal) needed for the development of an ethnomathematics research. The researchers have observed that this aspect has been theorized in anthropological research for decades, having the emic, etic, and dialogic approaches as the means to provide a deeper understanding of the mathematical ideas, procedures, and practices developed in diverse contexts. Our analysis reveals that innovative pedagogical actions need to cope with existing mathematics curricula, in order that mathematical ideas, procedures, and practices can be understood. This article aims at contributing to a holistic understanding of the mathematical ideas, procedures, and practices developed by members of distinct cultural groups.

Keywords: ethnomathematics; pedagogical action; responsible subversion.

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Resumo

Etnomatemática e a subversão responsável de sua ação pedagógica: uma investigação apoiada em três abordagens antropológicas

Este artigo teórico tem como principal objetivo refletir sobre os diversos pontos de vista abordados em nossa pesquisa em curso sobre a etnomatemática e suas conexões com a subversão responsável de sua ação pedagógica. Discutimos ideias, procedimentos e práticas matemáticas relacionadas às abordagens antropológicas — êmica (local/de dentro) e ética (global/de fora) — a fim de compreender a abordagem dialógica (êmica-ética/glocal), que é essencial ao desenvolvimento de pesquisas em etnomatemática. Os pesquisadores identificaram que, por décadas, esse aspecto tem sido teorizado em pesquisas antropológicas, contando com as abordagens êmica, ética e dialógica como fonte de um maior entendimento das ideias, dos procedimentos e das práticas matemáticas desenvolvidas em contextos diversos. Nossa análise revela uma necessidade de cooperação entre as ações pedagógicas inovadoras e o currículo matemático existente para que ideias, procedimentos e práticas matemáticas sejam compreendidos. Esperamos que este artigo contribua para um entendimento holístico das ideias, procedimentos e práticas matemáticas desenvolvidas por membros de grupos culturais distintos.

Palavras-chave: ação pedagógica; etnomatemática; subversão responsável.

Resumen

Etnomatemática y la subversión responsable de su acción pedagógica: una investigación apoyada en tres enfoques antropológicos

Este artículo teórico tiene como principal objetivo reflexionar sobre los diversos puntos de vista abordados en nuestra investigación en curso sobre la etnomatemática y sus conexiones con la subversión responsable de su acción pedagógica. Discutimos ideas, procedimientos y prácticas matemáticas relacionadas con abordajes antropológicos –émica (local/desde adentro) y ética (global/desde afuera)– con el objetivo de comprender el abordaje dialógico (émica-ética/glocal) que es esencial para el desarrollo de investigaciones en etnomatemática. Los investigadores identificaron que, por décadas, este aspecto ha sido teorizado en investigaciones antropológicas, contando con los enfoques émico, ético y dialógico como fuente de un mayor entendimiento de las ideas, de los procedimientos y de las prácticas matemáticas desarrolladas en contextos diversos. Nuestro

análisis revela una necesidad de cooperación entre las acciones pedagógicas innovadoras y el currículo matemático existente para que las ideas, procedimientos y prácticas matemáticas sean comprendidos. Esperamos que este artículo contribuya a un entendimiento holístico de las ideas, procedimientos y prácticas matemáticas desarrolladas por miembros de grupos culturales distintos.

Palabras clave: acción pedagógica; etnomatemática; subversión responsable.

Initial Remarks

Ethnomathematics opposes a dominant and Eurocentric discourse in mathematics education, which emphasizes the school curricula developed by colonizing countries and imposed on local communities during the process of colonization. Likewise, as a research program, it challenges the notion that members of local and/or distinct cultural groups develop only exotic and/or simplistic mathematical ideas, procedures, and techniques.

Thus, the development of an ethnomathematics program can be interpreted, to some extent, as a reaction to the *cultural imperialism* world-widely spread during the expansion of the Great Navigations in the beginning in the fifteenth century (D'Ambrosio, 1985). Such reaction possibly connects to the concept of *responsible subversion*[1] (Hutchinson, 1990) which, in turn, connects to the flexibility of rules and regulations, directed at achieving a deeper understanding of what constitutes mathematical thinking and reasoning.

In the field of education, according to D'Ambrosio and Lopes (2015), the concept of subversion refers to the practices of researchers and educators that insubordinately, though reasonably, oppose prescriptions with no pedagogical sense of educational bureaucracy and public policy. This concept also encompasses actions assumed in relation to the norms and institutional rules aimed at the needs of the school population.

Likewise, school administrators, educators, and teachers will be seen as responsible subversives if the can offer creative alternatives, producing better results for the common good of the school community – namely, their colleagues, as well as students and their parents. These actions are often taken in opposition and, generally, in challenge to the established authority, even if they can be traced back to unintentionally excluding and/or discriminatory policies.

For example, there is a large variety of mathematical procedures and techniques that oppose the *primitivist's*[2] belief that members of distinct cultural groups have been displaying, thus far, only simplistic mathematical knowledge to face the problems in their communities. These procedures and techniques also challenge the epistemological stereotypes most damaging to these individuals (Eglash et al., 2006).

[1] The concepts of creative insubordination (Crowson; Morris, 1982), responsible subversion (Hutchinson, 1990), and positive deviance (Zeitlin, Ghassemi; Mansour, 1990) are equivalent as they relate to the adaptability of rules and regulations in order that the welfare of the members of distinct cultural groups can be achieved.

[2] Primitivism refers to cultures believed to lack cultural, technological, or economic sophistication, or development; it has been used historically to justify conquer members of other cultural groups. The word primitive generally refers to someone or something not as complex or as advanced as the people or cultures to which they are being compared. It is conventionally defined under negative terms, as they are lacking in elements such as organization, refinement, and technological accomplishments. In cultural terms, this means a deficiency in those qualities that have been used historically in the West as indicators of civilization (Rhodes, 1995).

In this regard, responsible subversion poses as an important source of an adaptive-transformational capacity for members of distinct cultural groups; it produces non-conformism and opinion deviance (Dehler; Welsh, 1998). Moreover, it primarily aims to modify norms and rules by promoting innovation, creativity, and adaptability (Rosa; Orey, 2017).

Thus, responsible subversion means that subversive researchers and educators will develop awareness of when, how, and why they should stand against any established procedures or guidelines that present as unjust, racist, homophobic or unfavorable to members of the school community.

Being subversively-responsible requires assuming all members are unfinished beings who take creativity, criticality, responsibility, and curiosity as the foundation of knowledge building, and who make it an unfinished and permanent search movement (D'Ambrosio; Lopes, 2015).

Stemming from this context, ethnomathematics can be considered a subversive and responsive program because it causes a certain disruption to the existing order in the academic mathematics by encouraging and developing the study of the mathematics found locally, including diverse mathematical ideas, procedures, and practices that concord to the *emic*[3] perceptions of its members (Rosa; Orey, 2016).

In this regard, Rosa and Orey (2014) state that much of the antipathy towards ethnomathematics as a program is precisely because it has defied the rules and the bureaucratic expectations of traditional academic mathematics; it aims to recognize the different and diverse modes through which mathematical knowledge is built in other cultural environments.

At the level of responsible subversion, Rosa and Orey (2017) argued that, in the development of mathematical knowledge, it is necessary to reclaim the contributions of those who have been conquered or marginalized, or who pertain to minority groups. Ethnomathematics elicits a new respect for diverse forms of mathematical knowledge and points a way out of ethical dilemmas brought up by investigations in this area of study.

Thus, what makes responsible the subversion triggered by this program is the disturbance it has initiated; a turmoil that triggers a review of the Western, traditional academic mathematical knowledge by potentially increasing and creating new opportunities to debate the nature of the mathematics curriculum. This is not to say that the traditional mathematics is wrong: it is just not enough.

Still according to Rosa and Orey (2015), responsible subversion contributes to the challenging of taboos or to the outright hostility towards the suggestion that mathematics is a *universal*[4] field of study freed of traditions and cultural roots. Thus, D'Ambrosio (1990) affirms that mathematical knowledge is gained through uneven cultural interactions and conflicts, which reflects the dynamics of many cultural encounters.

Moreover, this theoretical article shares some reflections on the feedback collected from our ongoing work in ethnomathematics and its connections to the responsible subversion of its pedagogical action. Likewise, there is a discussion on the employed mathematical ideas, procedures, and practices that were drawn from anthropological approaches, such as emics (local/insiders) and etics (global/outsiders), in order to propel and understand the dialogic (emic-etic/glocal) approach that the development of an ethnomathematics investigations demands.

[3] The emic and etic approaches were developed by Pike (1967) from a distinction in linguistics between phonemic and phonetic. In their original meanings, phonemics refers to the examination of sounds for their meaning-bearing roles in a particular language, while phonetics denotes the study of universal sounds overarching all languages.

[4] Mathematics can be considered universal to the extent that members of all distinct cultural groups are able to order, count, pattern, solve problems, and model. However, the way in which these members, outside of the academic world, perform these activities pertains to the diversity that ethnomathematics seeks to study, share, and promote.

Ethnomathematics and its three approaches to viewing cultures

For researchers and educators, there is a challenge in accepting connections between mathematics and culture: it is the development of pedagogical actions that could facilitate the comprehension of the culturally-bound mathematical ideas, procedures, and practices developed by members of distinct cultural groups. All of which without letting their own culture interfere with the cultural background of these members.

Likewise, members of distinct cultural groups developed an insider's interpretation of their own environment — we call it *local* culture (*emic* approach) — as opposed to the outsider's perspective of it — the *global* interpretation (*etic* approach) (Orey; Rosa, 2014). The use of emics and etics to interpret cultural systems, including cognitive, perceptual, and conceptual knowledge, is *dynamic*[5].

Both emic and etic approaches suggest a way of discriminating between the various types of knowledge for the study of cultural phenomena, such as the development of mathematical practices. Thus, Pike (1967, p. 37) affirms:

> (. . .) it proves convenient – though partially arbitrary – to describe behavior from two different standpoints, which lead to results which shade into one another. The etic viewpoint studies behavior as from outside of a particular system, and as an essential initial approach to an alien system. The emic viewpoint results from studying behavior as from inside the system.

The emic approach examines the native (local) principles of classification and conceptualization from within each cultural system (Berry, 1989), in which distinctions made by the members of specific cultural groups are emphasized.

According to Lett (1990), the emic approach is essential for an intuitive and empathic understanding of a culture; while the etic approach is essential for a cross-cultural comparison and indispensable for ethnology because such comparisons necessarily demand the application of standard units and categories.

It is necessary to deconstruct the notion of mathematical ideas, procedures, and practices as uniquely modern or European in origin simply because they are based on values and certain philosophical assumptions strongly endorsed by Western civilizations and by science. For example, on one hand, Rosa and Orey (2017) assert that there are those who believe that mathematical procedures are unique, and that the sociocultural unit of operation is the individual. On the other hand, others believe that mathematical practices are the same and that their goals and techniques are equally applicable across all cultural groups.

An important goal is to challenge and strengthen the mathematics curricula already in place, in both their assumptions of mathematical universality and their claims of descriptive, predictive, and explanatory adequacy (Rosa, 2010). A second goal is to understand and explain both current and historical varieties of mathematical ideas, procedures, and

[5] Cultural dynamism refers to the exchange of systems of knowledge that facilitate members of distinct cultures to exploit or adapt the world around them. Thus, cultural dynamics enables the incorporation of human inventions, which relates to the idea of changing the world in order to create new abilities and institutionalizing these changes. In turn, this serves as basis for the development of more competencies (Rosa; Orey, 2015).

practices that change across time, culture of origin, race, ethnicity, gender, and other sociocultural characteristics (Orey, 2015).

In agreement with this discussion, three *cultural views* or approaches that help us investigate mathematical ideas, procedures, and practices developed by the members of distinct cultures were identified:

1. *Etic* (*global/outsider*) is the outsiders' view on beliefs, customs, and on the scientific and mathematical knowledge of the members of distinct cultural groups. In this context, global analyses follow a cross-cultural approach because outsiders develop global worldviews that seek objectivity across cultures. Thus, Helfrich (1999) addresses the matter of a cross-cultural perception in which observations are often made in compliance with externally-derived criteria, and frequently with no intention of learning the perspectives of others. With globalization, the Western bias and the utilitarian approach to school mathematics have been reinforced in the prevailing mathematics curricula, and it has helped to widen the acceptance of pervasive mathematical ideologies. School mathematics, in particular, has been criticized as a cultural-homogenizing force, a critical filter for status, a perpetuator of mistaken illusions of certainty, and an instrument of power (Skovsmose, 2000). In this approach, comparativist researchers and educators attempt to describe differences among cultures. Then individuals are considered as *culturally universal* (Sue; Sue, 2003). In this context, Pike (1967) refers to etic categories as culture-free features of the real world.

2. *Emic* (*local/insider*) is the insiders' view on their own culture, customs, beliefs, and on the scientific and mathematical knowledge. Local knowledge matters because it has been tested and validated within the local context. It creates a framework from which members of distinct cultural groups can understand and interpret the world around them. Moreover, the worldviews of locals shed light on intrinsic cultural distinctions that examine native principles of classification and conceptualization from within each cultural system. Currently, the importance of the local contributions to the development of scientific and mathematical knowledge is recognized. For example, local mathematical knowledge and interpretations are essential to emic analyses in the mathematics curriculum, which cultivates values and fosters *conscientization* among students. An emic analysis is culturally specific in regard to the insiders' beliefs, thoughts, behaviors, knowledges, and attitudes. It is from their viewpoint that mathematical knowledge is conveyed for the understanding of their cultural context. In this approach, the members describe their culture in their own terms and they are considered as *culturally specific* (Sue; Sue, 2003). In this context, Helfrich (1999) stated that it is the self-determination

and self-reflection of these members in regard to the development of their mathematical ideas, procedures, and practices that should take emphasis in this approach.

3. *Dialogic* (*glocal/emic-etic*) is the product of the continuous interaction between etic (globalization) and emic (localization) approaches; which offers a perspective of these that are elements of the same phenomenon (Kloos, 2000). It involves blending, mixing, and adapting two processes in which one component must address the local culture and/or the system of values and practices (Khondker, 2004). In a glocalized society, members of distinct cultural groups must be "empowered to act globally in its local environment" (D'Ambrosio, 2006, p. 76). It is also necessary to work with different cultural environments and, acting as ethnographers, to describe mathematical ideas, procedures, and practices of other peoples in order to give meaning to these findings (D'Ambrosio, 2006). Therefore, Rosa and Orey (2017) argued that glocalization has emerged as the new standard when it comes to reinforcing positive aspects of worldwide interaction in textual translations, localized marketing communication, socio-political considerations, and in the development of scientific and mathematical knowledges.

By initially focusing on the local knowledge to then integrating global influences, it is possible to create individual and collective groups, rooted in their local cultural traditions and contexts but also equipped with a global knowledge; all of which is done through the creation of a sort of localized globalization (Cheng, 2005).

For example, emic-oriented researchers and educators focus on investigating the intrinsic cultural distinctions that are meaningful to the members of distinct cultural groups; they seek to understand whether the natural world is distinguished from the supernatural realm in those specific cultures' worldview (Rosa; Orey, 2017).

Contrariwise, etic-oriented researchers and educators examine cross-cultural perspectives; by doing so, they make observations in accordance to externally-derived criteria. This scenario allows for the comparison of multiple cultural groups, in which "both the objects and the standards of comparison must be equivalent across cultures" (Helfrich, 1999, p. 132).

Thus, should researchers and educators accept the imposed cultural *universality*[6] (global) of mathematical knowledge or should they take on techniques, procedures, and practices of *cultural relativism*[7] (local)? Researchers and educators seeking to merge universal (global) and community specific (local) approaches will face the classic dilemma of scientific goals conflicting with investigations in ethnomathematics.

Conversely, by using both emic and etic approaches, one's understanding of important issues in ethnomathematical scientific researches and investigations deepens due to the fact that these are complementary worldviews (Rosa; Orey, 2013). As such, they enable an outlining of forms of synergy between local and global aspects of mathematical knowledge.

[6] Cultural universality refers to the belief that the origin, process, and manifestation of disorders are equally applicable across cultures (Bonnett, 2000).

[7] Cultural relativism is related to the assertion that human values, far from being universal, vary according to different cultural perspectives in distinct cultures. Individuals' beliefs, values, and practices are understood based on their own culture, rather than judged against the criteria of another (Tzvetan, 1993).

To deal with this dilemma, it is advisable to combine the emic and etic (local-global) approaches, rather than simply applying the local or global dimensions of one culture to another. This combined approach requires of researchers and educators some familiarizing with the local knowledge developed by the members of distinct cultural groups; this allows one to become acquainted with the relevant cultural differences in diverse sociocultural contexts (Rosa; Orey, 2015).

Both local (emic) and global (etic) approaches are important for the development of mathematics education, most prominently in the conduction of research in ethnomathematics. In this context, local knowledge and its interpretations (emic) are essential, as well, to the debates around the comparisons of it to the mathematical knowledge developed in distinct cultural contexts (etic). In this regard, Pike (1967, p. 41) stated that:

> Through the etic 'lens' the analyst views the data in tacit reference to a perspective oriented to all comparable events (whether sounds, ceremonies, activities), of all peoples, of all parts of the earth; through the other lens, the emic one, he views the same events in that particular culture, as it and it alone is structured. The result is a kind of 'tri-dimensional understanding' of human behavior instead of a 'flat' etic one.

In sum, Pike's (1967) views the relation between the *emic* and *etic* approaches as a *symbiotic* process between two different mathematical knowledge systems and also as something extremely valuable. Similarly, the resurgence of debates on cultural diversity in the mathematics curriculum has also renewed the classic emic-etic debate; since one needs to comprehend how to build scientific generalizations while understanding and making use of sociocultural diversity.

Nonetheless, attending to unique mathematical interpretations developed by the members of each cultural group often challenges the fundamental goals of mathematics; in which the main objective is to build a theoretical basis that describes the development of mathematical practices in distinct cultures.

Aspects of responsible subversion in the ethnomathematics as a program

Researches and investigations in ethnomathematics have looked into historical accounts to reveal the cultural influences in the evolution of mathematical knowledge. For example, Orey (2000) argues that this approach assists in the analyses of mathematical ideas, procedures, and practices developed locally aimed to deconstruct the dominant mathematical discourse by proposing innovative views on the nature of this knowledge.

In this regard, responsible subversion is employed because the norms and rules applied in academic mathematics are inconsistent with the mathematical knowledge developed in the local context. For example,

Rosa (2010) emphasizes that investigations in mathematics education have overlooked connections between academic mathematical knowledge and the practices developed locally by members of distinct cultural groups.

To reduce the gap between the theoretical and the practical mathematical knowledge, there is a need for researchers and educators to query about possible connections between the mathematical knowledge developed in particular cultural contexts and that practiced and supported by academic institutions (Rosa, 2010).

The responsible subversive aspect of ethnomathematics as a program recognizes both the uniqueness and the perspectives of members of distinct cultural groups by emphasizing their emic mathematical knowledge systems and by valuing them on their own terms and contexts (Rosa; Orey, 2016).

Ongoing investigations in ethnomathematics must describe the ideas and procedures that are implicit to the mathematical practices developed locally. In this sense, Lloyd (2011) affirms that research on these practices can be regarded as a form of resistance towards the imposition of academic mathematical knowledge, as it suggests possible measures in search of creative and innovative solutions to these challenges.

For example, there was a study, conducted by Duarte (2004) in Brazil, that dealt with the specificity of mathematical ideas, procedures, and practices carried by adolescent and adult construction workers taking an evening adult education course. This study found that the mathematical knowledge produced, developed, and transmitted in construction sites had important curricular implications.

Furthermore, Duarte (2004) also studied the connections between local and academic knowledge in order to establish curricular modifications. He found that these connections had a positive influence over the development of a mathematics curriculum in schools. In our point of view, the responsible subversion found in the ethnomathematics as a program contributed to the assembly of a new mathematical knowledge.

Through the investigations, while seeking to comprehend the development of local mathematical knowledge, researchers and educators may face specific aspects related to ideas, procedures, and mathematical practices that differ from those studied in the academy (Rosa; Orey, 2012). This aspect of responsible ethnomathematics subversion can help to solve ethical dilemmas evoked by the investigations in this program.

In this respect, the mathematical knowledge must be interpreted in a broader sense, given that *ethno* relates to members of identifiable cultural groups, such as national and tribal societies; working groups; given-aged children; individuals belonging to distinct professional classes, and marginalized/minority cultural groups (D'Ambrosio, 1985).

For example, Pinheiro (2017) proposed an innovative ethnomathematics pedagogical initiative for the teaching and learning of mathematics by deaf people. The results of his investigation have shown that it is necessary to promote the development of academic and professional skills for deaf students through inclusion and access, especially, in regard to financial education content.

Another important contribution given by Pinheiro's (2017) study came in the form of respect and attention shown for deaf's culture and their daily experiences; this was in regard to responsible ethnomathematics subversion and related to the development of the financial education of the students who communicate in Brazilian Sign Language (Libras). This particular pedagogical action was relevant to promote a significant relation between the day-to-day knowledge and the one systematized by the school.

In his study, he employed a methodology linked to the contextualization of everyday phenomena; through which it was possible to negotiate meaning, thus, favoring the developing of mathematical and financial concepts with deaf students (Pinheiro, 2017). This discussion reveals the need for both researchers and educators to part with the broader Western-Eurocentric perspective on the process of development of the mathematical knowledge (Anderson, 1997).

In other words, responsible subversion in mathematics education refers to behavioral, cultural, political, economic, environmental, and social changes premised on the observation that, in any given context, members of distinct cultural groups, when faced with similar challenges, will employ uncommon but efficient mathematical ideas, procedures, and strategies to solve the problems in their own communities (Rosa, 2015).

According to Bjelland *et al*. (1990), researchers and educators assist this ongoing reconstruction process, which relates mathematical knowledge with sociocultural activities through the use of descriptive indicators of distinct cultures, namely:

- *Artifacts* which are observational objects, material items, and technologies created and developed by members of distinct cultural groups. These instruments provide information about their makers and users. They meet basic necessities as well as provide recreation and entertainment; some examples of them include buildings, tools, means of transportation, musical instruments, and computers (D'Ambrosio, 2006). The technological subsystem made of the material objects is also perceived as artifacts along with the techniques to their usage, by means of which people are able to live. These tools and other instruments enable members of distinct cultural groups to feed, clothe, house, defend, transport, and amuse themselves. Protection from the elements, self-defense and eating are basic necessities. Huxley (1955) called artifacts the material objects used by these members to attend to those needs.

- *Mentifacts* are analytical tools, including thoughts, reflections, concepts and theories that represent ideas and beliefs of members of a particular cultural group. They also are shared ideas, values, and behaviors in a culture. Some examples of this category include religions, languages, viewpoints, worldviews, laws, and the concept of what is a right or wrong behavior (D'Ambrosio, 2006). They constitute the ideological subsystem that consists of ideas,

beliefs, and knowledges of a culture as well as of the ways in which they are expressed in speech or other forms of communication. Mythologies and theologies, legend, literature, philosophy, and folk wisdom are also part of this category. Passed on from generation to generation, these abstract belief systems tell members of distinct cultural groups what they ought to believe or value, and how they ought to act. Huxley (1955) argues that beliefs form the basis of the socialization process.

- *Sociofacts* represent the social structure of members of distinct cultural groups, such as family, tribal structures, and organizations that influence social behavior. They make the patterns of interpersonal relations expected and accepted among the members of these groups. Sociofacts include families, governments, educational systems, sports organizations, religious groups, and any other gathering designed for specific activities (D'Ambrosio, 2006). Thus, the sociological subsystem of distinct cultural groups is the sum of the expected and accepted patterns of interpersonal relations that find their outlet in economic, political, military, religious, kinship, and other associations. Sociofacts often define the social organization in order to regulate how members of distinct cultural groups function in relation to the group, whether it be family, church, or state. In what concerns the patterns of interaction in any of these associations, there are no givens, except that most cultures cater a variety of formal and informal ways of structuring behavior. This context enabled Huxley (1955) to state that differing patterns of behavior are learned and passed on from one generation to another.

Members of distinct cultural groups, in their search for transcendence and survival, elaborate explanations for the problems they face; they gather information that fuel the creation of their own myths and mysteries to help them explain, as mentifacts, their sociocultural and natural environments (D'Ambrosio, 2011).

Material representations of the reality (artifacts), organized in the form of language, art, and techniques, are observed and interpreted by members of other cultural groups; in this process, codes, symbols, and representations are created. Mentifacts, which are shared by members of distinct cultural groups through the use of diverse artifacts, constitute the sociofacts of their cultures (D'Ambrosio, 2011).

Mathematical artifacts (practices and tools) and mentifacts (aims, objectives, concepts, and ideas) are first developed by members of distinct cultural groups trying to cope with natural, social, economic, political, and cultural environments and to deal with situations and problems. They also attempt to explain and understand mathematical facts and phenomena that occur in their day-to-day life. These *ad hoc* artifacts and mentifacts are organized, transmitted, diffused, and shared with others (D'Ambrosio, 2011).

Artifacts, mentifacts, and sociofacts meet their objectives by being useful to the sociocultural development of the members of distinct cultures. Likewise, they will be shared and acquired by others in the communities and society as part of the sociofacts of these cultures. Hence, D'Ambrosio (2011) argued that it is important to highlight that the concept of cultures introduced by Huxley (1955) contemplates the notion of *artifacts+menti facts+sociofacts*.

It is also important to highlight that, according to Rosa and Orey (2015), the responsible subversion aspect of this approach relates to the use of creative insubordination in ethnomathematics as a program to tend to the students' learning process through the use of artifacts, mentifacts, and sociofacts, linked to *cultural traits*[8] of their own cultures.

Hence, through a pedagogical action applied to the daily life of members of the school community under study (Rosa; Orey, 2015) — who develop and use their own artifacts (products), sociofacts (practices) and mentifacts (perspectives) (Fantini; Fantini, 1997) — activities are shared in the classroom by members of distinct cultural groups in the elaboration of mathematics curricula (Rosa, 2010).

In this context, Moran (2001) affirms that artifacts are products made by members of distinct cultural groups, whether tangible (buildings, clothes, and works of art) or intangible (music and poetry). Sociofacts are practices, actions, and interactions of these members, including verbal and nonverbal communication, customs, and rituals. Mentifacts are the perspectives, thoughts, beliefs, values, and attitudes that underlie the conjunction of the other two cultural descriptive indicators.

Samples of these indicators are identifiable in the case of distinct cultures, however, a microcosm of the same pattern can be observed in self-selected cultural groups. For example, the results of Cortes' (2017) study show that farmer vendors have their own artifacts (manual scales and different packing products), mentifacts (mental calculations, distinct ways of pricing products, distinct ways of weighing products) and perspectives (trust in clients and in other farmers, and the common goal of selling good products at reasonable prices). In this context, all of these characteristics form a bond among the members of a distinct cultural group, such as farmer and vendors.

This perspective aims to reduce prejudice, inequity, and harm due to further disconnections between the mathematical knowledge, as practiced in the academy (etic), and its practical uses in everyday life (emic). For Rosa and Orey (2016), this means that the conduction of research in ethnomathematics is seen as a responsible subversion that uses the theoretical and methodological apparatus of these investigations to bring up and combat the privilege and the authoritarianism that has been granted to the academic mathematical discourse.

In this context, the results of Cortes' (2017) study, carried in a public school of Minas Gerais and in a local farmers' market[9], showed how the dialogical approach to ethnomodeling adds to the process of re-signification of function concepts.

[8] Cultural traits relate to the appreciation of features developed by the members of a specific culture — such as religion, language, government, customs and traditions, social organization, and arts — as well as the establishing of relationships between the members of that group. In this context, cultural systems are composed of material and immaterial traits that are tangible and intangible items. They are the identifiable elements of a cultural system. Groups of similar traits are components or sub-systems of a culture. Cultural traits can be organized into artifacts, mentifacts, and sociofacts. Thus, it is important for researchers and educators to understand the cultural roots of other cultures in order to value the ideas, procedures, and mathematical practices used by students from distinct cultural contexts (Rosa; Orey, 2016).

[9] The local farmers' market, where this research was carried out, sells food, horticultural products, clothing, and handicrafts. However, this study dealt only with the labor practices of a marketer in connection to the commercialization of horticultural products (Cortes, 2017).

His study produced data on 38 students in the second year of high school during their interactions with a farmer and his labor practices. Fitzsimons (2003) stated that this approach enables the comprehension of how privilege and authoritarianism — stemming from the colonization — have influenced the distribution of power in modern society.

Cortes (2017) also stated that his specific objectives were: a) to describe the connection between ethnomathematics and mathematical modelling; b) to create an understanding of the importance of cultural concepts for the development of mathematical ethnomodels extracted from daily practices found in the sociocultural context of the students in this study; c) to describe how the local approach (emic), the global (etic) approach, as well as the subsequent glocal (dialogical) approach to the ethnomodeling process appear in interactions and meetings between the student group and the farmer; d) to verify how the mathematical practices of the farmer were adapted and used in the classroom to develop educational actions in relation to the connection between ethnomathematics and mathematical modelling through ethnomodeling.

According to Cortes (2017), one of the main results of this study is that ethnomodeling provided an integrative approach to the school curriculum that takes into consideration the origins of both the etic and the emic mathematical knowledge, in such a way that teachers and students come to understand, more realistically, holistically and comprehensively, the mathematics practiced by members of the different cultural groups that make up diverse student populations.

Final remarks

The discussions in this article have revealed analogous uses for responsible subversion in the conducting of research in ethnomathematics. For example, mathematical thinking is developed in distinct sociocultural contexts with specific needs and ways of life. It is important to analyze the relation between culture and mathematics, questioning the predominant view that mainstream mathematics is culture-neutral.

However, it is necessary for both researchers and educators to be willing to, and supported in, taking risks associated with the choosing of the local mathematical knowledge to feature in mathematics curricula. This decision-making process is one of the most important aspects of responsible subversion (Haynes; Licata, 1995).

Furthermore, Rosa and Orey (2016) comment that this approach can be understood as a stand against the dehumanizing effects of the bureaucratic authoritarianism displayed in the researches related to ethnomathematics as a program.

This article has outlined our ongoing research related to cultural perspectives in mathematics; a work that acknowledges that contemporary academic mathematics is predominantly Eurocentric and Anglo-centric.

Eurocentrism and Anglo-centrism have ignored, hindered and even destroyed local mathematical ideas, procedures, and practices; specifically, those once practiced by vanquished societies and cultures. Many procedures and practices birthed in local traditions, and that represent an emic approach, have been lost; many are considered inferior, and therefore unimportant (Rosa; Orey, 2015).

There is no point in doubting the importance of modern science and mathematics; yet, westernized mathematics has come to be primarily dominated by the capitalistic preferences of the European and North-American science and accompanying Eurocentrism, which poses many problems for the mathematics education in both non-Western and/or non-dominate cultures. For instance, there is D'Ambrosio's (2006) statement that conceptions of mathematical practices have been imposed globally, through a series of colonial intrusions, in accordance to the pattern of rational human behavior.

By adopting systematic studies in an ethnomathematical perspective, we can develop new contexts, skills even, that allow us to observe mathematical phenomena on a more inclusive and broader wave length. The results may then lead to new viewpoints in the mathematics education capable to improve cultural sensitivity in teaching practices.

In this regard, we see ethnomathematics as the study of mathematical phenomena within a culture; which differs from the traditional conception that takes it as the foundations of the one kind of mathematics that is a constant and it is applicable to everyone and everywhere. For being culturally bound, mathematics then becomes a social construct.

This article discussed concepts of responsible subversion from the perspective of ethnomathematics as a program. The pedagogical action of this program helps students to overcome the use of disassociated techniques and formulas that are often blindly memorized; it also allows them to develop strategies to get access to diverse mathematical representations in a new formative dimension of the mathematical nature.

D'Ambrosio's (1985) ethnomathematics program emphasizes the importance of the community for a school, as it seeks to connect school practices to the mathematics developed locally. It is necessary for the school curriculum to be designed to value and promote local knowledge (emic) and practices developed by members of the communities who are part of their school contexts.

This perspective provides a necessary balance to school curricula. The integration of these components in the mathematics curriculum facilitates the conceptions of ethnomathematics as a program and aims at the humanization of mathematics through contextualized approaches in the ongoing curriculum development. This is one of the most important responsible subversion characteristics in any pedagogical action.

In conclusion, it is important to emphasize action in relation to the context of an ethnomathematics as a program that enables a comprehensive analysis of the school context. Pedagogical practices transcend physical

environments aiming to welcome knowledges and procedures developed in the diverse sociocultural contexts of students (Rosa; Orey, 2015).

In this approach, one important pedagogical proposal for the development of the school curriculum relates to the transformation of mathematics into a living knowledge that integrates real situations through questionings, analysis, and critical reflection of the phenomena that occur in the everyday life of learners.

It is in the school community itself that researchers and educators will easily find the didactic elements of the mathematical content needed to develop a mathematics curriculum (D'Ambrosio, 2006). There is a need to diversify teaching strategies used in the mathematics curriculum, as well as the use of ethnomathematics as a program in schools.

However, we take this opportunity to acknowledge that there is not a single formula for improving students' performance in mathematics. Educators need to be supported as they develop processes which will allow them to implement innovative pedagogies to help students to reach their sociocultural and academic potential.

Responsible subversion, especially when it comes to ethnomathematics, can be taken as a tool to combat the dehumanizing effects of curricular and bureaucratic authoritarianism, and as a tool for peace. In this context, Haynes and Licata (1995) argued that this subversion aims to ensure that curricular bureaucracies do not disservice students when public policies and institutional procedures have no real connections with school communities.

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